Additive Modeling of Functional Gradients

Fang Yao and Hans-Georg Müller
University of Toronto
Department of Statistics
100 St. George Street, Toronto, ON M5S 3G3, Canada
fyao@utstat.toronto.edu

Abstract

We consider the problem of estimating functional derivatives and gradients in the framework of a functional regression setting where one observes functional predictors and scalar responses. Derivatives are then defined as functional directional derivatives which indicate how changes in the predictor function in a specified functional direction are associated with corresponding changes in the scalar response. Aiming at a model-free approach, navigating the curse of dimension requires to impose suitable structural constraints. Accordingly, we develop functional derivative estimation within an additive regression framework. Here the additive components of functional derivatives correspond to derivatives of nonparametric one-dimensional regression functions with the functional principal components of predictor processes as arguments. This approach requires nothing more than estimating derivatives of one-dimensional nonparametric regressions, and thus is computationally very straightforward to implement, while it also provides substantial flexibility, fast computation and asymptotic consistency. We demonstrate the estimation and interpretation of the resulting functional derivatives and functional gradient fields in a study of the dependence of lifetime fertility of flies on early life reproductive trajectories.